FIN-408 **Stochastic calculus I**

Malamud Semyon

Cursus	Sem.	Type
Ing. finance	MA1, MA3	Obl.

Language	English
Credits	4
Session	Winter
Semester	Fall
Exam	Written
Workload	120h
Weeks	14
Hours	**4 weekly**
Lecture	2 weekly
Exercises	2 weekly

Remarque

For sem. MA1

Summary

This course is an introduction to probability theory and stochastic calculus. It starts with basic notions of probability, characteristic functions and limit theorems. Then, we study stochastic processes and martingales in discrete and continuous time, including Brownian motion and Ito calculus.

Content

1. Probability review (4 weeks) Probability spaces - sigma algebras - random variables - probability measures - independence - Jensen inequality and other basic inequalities for expectations - law of large numbers - central limit theorem - large deviations

2. Discrete time processes (4 weeks) Random walks - Markov chains - calculations with stopping times - filtrations - martingales - Gaussian distributions and discrete time Kalman filtering

3. Continuous time processes (3 weeks) Brownian motion - continuous filtrations - Gaussian processes - Kolmogorov's theorem - martingales - convergence - optional sampling - Levy's theorem - Doob's theorems - quadratic variation

4. Stochastic calculus (3 weeks) Ito's integral - Ito's isometry - Ito's formula - Ito's processes - stochastic differential equations

Keywords

Stochastic calculus, probability

Learning Prerequisites

Important concepts to start the course

Basic analysis, some understanding of probability

Learning Outcomes

By the end of the course, the student must be able to:

- Work out / Determine moment generating functions, conditional moment generating functions, conditional and unconditional moments for multi-dimensional random vectors. Apply the Law of Large Numbers and the Central Limit Theorem.
- Analyze multi-dimensional Gaussian distributions and derive the corresponding conditional expectations and conditional variances.
- Apply Kalman filter to a general linear model and derive the filter and the optimal Kalman gain.

- Work out / Determine basic properties and moment generating functions of stopping times for general random walks and Markov chains. Derive the martingale representation property for binomial filtration.
- Derive basic properties of Brownian motion and the corresponding martingales. Formulate Levy Theorem and its implications. Describe Brownian motion as a continuous time limit of a random walk (Donsker'## theorem).
- Operate Ito formula and use it to derive useful properties for any given function of a multi-dimensional Ito process.
- Describe an Ornstein-Uhlenbeck process, derive its basic properties, and use it to compute expectations and transition densities, both for stationary and non-stationary processes.
- Apply Ito representation theorem and understand its link to market completeness.

Transversal skills

- Plan and carry out activities in a way which makes optimal use of available time and other resources.
- Use a work methodology appropriate to the task.
- Evaluate one's own performance in the team, receive and respond appropriately to feedback.
- Continue to work through difficulties or initial failure to find optimal solutions.

Teaching methods

Ex cathedra classes / exercise sessions

Assessment methods

20% continuous control
40% written mid-term exam
40% written final exam

Supervision

Assistants	Yes

Resources

Bibliography

R. Durrett, "Stochastic Calculus. A Practical Introduction", CRC Press, 1996. B. Øksendal, "Stochastic Differential Equations. An Introduction with Applications", Springer Verlag, 2003. S. Shreve, "Stochastic Calculus for Finance" (2 volumes), Springer Verlag, 2004. I. Karatzas and S. Shreve, Brownian Motion and Stochastic Calculus. Springer Verlag, 1998.

Ressources en bibliothèque

- Stochastic Calculus for Finance I / Shreve
- Stochastic Calculus for Finance II / Shreve
- Brownian Motion and Stochastic Calculus / Karatzas
- Stochastic Calculus / Durrett
- Stochastic Differential Equations / Øksendal

Prerequisite for

- Advanced derivatives
- Credit risk
- Derivatives
- Fixed income analysis
- Real options and financial structuring

- Stochastic calculus II